Imperfect Labor Market and Convergence:
Theory and Evidence for some OECD Countries

Gaetano Carmeci and Luciano Mauro

Department of Economics and Statistics, University of Trieste
Piazzale Europa, n. 1, 34127 Trieste, Italy
Phone: +39 0406767100; Fax: +39 040567543
E-mail: carmeci@mail.univ.trieste.it
Imperfect Labor Market and Convergence: Theory and Evidence for some OECD Countries

Abstract

In this paper we show the existence of a negative relationship between long run growth and labor market imperfections both theoretically and empirically. We consider a “monopolistic union” imperfect labor market in a neoclassical growth framework and show that labor market rigidity, captured by the mark-up over the reservation wage, does lower the growth rate along the transitional path. We verify empirically the convergence relationship implied by the model on a panel of 18 OECD countries using both traditional cross-countries growth regression and the system GMM estimator proposed by Arellano and Bond (1995) and Blundell and Bond (1998). The data support the basic implications of our model. In particular, the standard proxies for mark-up, the unemployment replacement ratio and union density, result to affect negatively the long-run growth rate. We also analyze the relationship between growth and an alternative mark-up proxy based on Bean’s insight (ibid.,1994): the ratio of per worker wage on per capita consumption. A negative and significant relationship between output growth rate and this mark-up proxy is found.

JEL-Code: O41, E24, C51,C52.
Key words: growth, unemployment, convergence, GMM estimation, dynamic panel data model.

1. Introduction and Relation to the Literature

The “old” issue of Long Run Growth and Unemployment has been the subject of several recent contributions (Aghion and Howitt (1994), Van Schaik and De Groot (1998), Daveri and Tabellini (2000), Peretto (2000) among others). Nevertheless, the links between unemployment and growth proposed in the literature are far from being univocal. Pissarides (1990), for example, considers the effects of an exogenous rate of productivity growth in a labor market model of the “search” type. He finds a positive relationship between productivity growth and the rate of job creation that causes a decrease in unemployment rate for a given flow of unemployed agents. An “inverted U” relationship between innovation and unemployment is derived in Aghion and Howitt (1991,1994) where the authors consider both the re-allocation of jobs that accompanies the innovation processes as well as the positive effect on capital accumulation due to innovations.

In the two preceding approaches the causality links are from growth to unemployment although it is natural to consider the opposite direction as well. Bean and Pissarides (1993) consider an overlapping generation growth model with an imperfect labor market where the higher the equilibrium unemployment the lower the pool of saving of the current generation that in turn lowers the growth rate of the economy. Tabellini and Daveri (2000), also in an overlapping generation context, consider a monopolistic union that is able to transfer all increases in labor taxes on firms. Firms observe decreasing returns to capital in trying to substitute costly labor factor and restrain capital investment lowering the growth rate of the economy.

Other authors explore the possible effects of unemployment upon human capital accumulation. Bean and Layard (1989) are the first to our knowledge to suggest the possibility of a deterioration of human capital due to an extended period of unemployment. This idea has been developed by Podrecca (1998) whose model implies a negative relationship between equilibrium unemployment rate and long run growth whereas during the transitional dynamics the two can be positively related depending on the parameters values. Saint Paul
(1991) instead in a stylized two sectors endogenous growth model inserts an “efficiency wage” mechanism that implies a positive relationship between unemployment and growth.

Connected to the innovation process and labor market imperfections as in Haghion and Howitt (1994) the model by Bertola (1994) imply a negative link between growth and unemployment. Firms in his economy operate in a set up a’ la Grossman and Helpmann (1991) or Romer (1990) and they are subject to idiosyncratic shocks due to product innovations calling for labor force re-allocations. To the extent that rigidity of labor market prevents firms from adjusting their labor factor, labor market imperfection causes a lower innovation rate and a higher unemployment rate. A positive relationship is found in Aghion and Saint Paul (1991), although their model belongs more appropriately to a business cycle approach. In fact, firms in their model find convenient to innovate the production processes during recession periods and therefore a positive association between productivity growth and unemployment is observed.

Van Schaik and DeGroot (1998) propose a two sectors R&D model where a trade off between growth and unemployment is obtained when more competitive environment is introduced in the high-tech sector. However an opposite result is obtained by Peretto (2000) who, in a somehow similar set up, underlines the scale effect in the innovative sector. In his model pro-flexibility Labor market reforms, reducing costs, ease new entry and enlarge the scale of the economy, fostering growth and decreasing the unemployment further.

As we have seen from the above short, and certainly partial review of the literature the proposed links between growth and unemployment are different and frequently opposite in sign.

In general the researchers turned their attention mostly toward endogenous growth models. This is probably due to the fact that the exogeneity of technological progress of the standard neoclassical growth model implies that long run growth rate and the rate of unemployment are independent, even in the presence of an imperfect labor market (Gordon, 1995).

Nevertheless the exogenous growth model has not been dismissed by the empirical growth literature which among other things underlined the importance of the transition dynamics and the convergence processes toward steady states and within economies (Temple (1999)). Turning our attention toward the transitional dynamics we are able to show that the independency between the long run growth rate and the rate of unemployment does not hold anymore. In fact in a neoclassical exogenous growth model with an imperfect labor market we show that labor market rigidity does influence negatively the growth of the economy along the convergence process. As long as convergence is a longtime process, labor market imperfections might be among the determinants of economic growth.

In the empirical section we test the empirical relationships implied by our theory on a panel of eighteen OECD countries. In particular, we estimate the convergence growth equations derived by the model both in a standard cross-countries growth regressions set up and in a dynamic panel data context using the system GMM estimator recently proposed by Arellano and Bond (1995) and Blundell and Bond (1998).

As implied by the model we found a negative impact on growth of standard measure of labor market imperfections such as: the unemployment replacement ratio and union density. We also analyze a model relationship between growth and an alternative imperfection proxy based on Bean’s (1994) insight. This
proxy, the ratio of per worker wage on per capita consumption, is found to negatively affect growth in accordance with the theory.

2. The Model

The economy is composed by N individuals each of them belonging to the labor force. Assuming a standard Cobb-Douglas with labor augmenting technical progress the output (Y) can be expressed in per capita terms (y) as a function of per capita capital (k) and the employment rate (l):

(1) \[ y = k^{\alpha} l^{1-\alpha} A^{1-\alpha}. \]

The labor market is of the “monopoly union” type where a myopic monopolistic union sets a wage and firms hire the labor quantity that equalizes the marginal product of labor to wage. The myopic behavior of the union that doesn’t discount future consequences of its present actions is a shortcut to keep the model tractable. However it is not such an implausible assumption as soon as one thinks of the problems of committing to future courses of action (see Daveri and Tabellini (2000)) and Unions are elective institutions suffering from the same problems of elected governments.\(^1\)

The union is supposed to maximize with respect to wage the expected utility of its representative member:

(2) \[ U(w)l(w) + U(\bar{w})(1-l(w)) \]

where \( w \) and \( \bar{w} \) is the wage and the reservation wage respectively. The utility function is of the CRRA type:

(3) \[ U(w) = \frac{w^{1-\theta}}{1-\theta} \]

and from equation (1) the demand for labor is:

(4) \[ l(w) = k \left( \frac{(1-\alpha)A^{1-\alpha}}{w} \right)^{\frac{1}{\alpha}} \]

Maximization of (2) subject to (3) and (4) implies the following well known condition:

(5) \[ w = \bar{w} (1-\alpha(1-\theta))^{\frac{1}{\theta}} \]

Equation (5) states that the wage is set as a mark-up over the reservation wage (Solow and McDonald (1981)). As far as the reservation wage, according to Bean (1994): “The reservation wage should include not only the real value of unemployment benefits, but also the value of leisure in consumption units.” The author, assuming a standard isoelastic utility function, shows that the reservation wage becomes a linear function of the level of per capita consumption (Bean (1994), footnote 2, pg. 527).\(^2\)

---

\(^1\) A complete inter-temporal imperfect labor market model although certainly useful should rely on some sort of heterogeneity in Unions’ preferences to explain the cross countries evidence on growth and unemployment. We think that a simpler model like the one we propose is able capture this mechanism.

\(^2\) As an example, using a Cobb-Douglas utility function of the kind: \( u = \left( \frac{\tilde{h} - h}{\tilde{h}} \right)^{-\alpha} \) where \( \tilde{h} \) is the time endowment and \( h \) the labour time, the standard equilibrium condition \( \frac{U}{P_c} = \frac{U}{(\bar{w} - L)} \) implies that \( \bar{w} = (1-\alpha) \frac{L}{\tilde{h}} \) (Bean (1994)b, footnote 2, pg. 527). In the model we retain Bean’s insights without considering explicitly the labour-leisure choice which would have complicated the model without any substantial gain.
Following Bean’s insight we as well assume a linear relationship between reservation wage and per capita consumption \(c\).

\[
\bar{w} = \beta c ,
\]

so that equation (5) becomes:

\[
w = \psi c
\]

where \(\psi = \beta \left(1 - \alpha (1 - \theta)\right)^{1/\theta} .\)

Finally, from (5) and (7) the equilibrium employment rate is:

\[
l = \left(\frac{1 - \alpha}{\psi}\right)^{\frac{1}{\alpha}} A^{\frac{1 - \alpha}{\alpha}} \frac{1}{c^{\frac{1}{\alpha}} k} = \left(\frac{1 - \alpha}{\psi}\right)^{\frac{1}{\alpha}} \frac{1}{c^{\frac{1}{\alpha}} k}
\]

where the hat over the variables implies that they are divided by the technology index \(A\).

We now consider the representative consumer problem whose objective function to maximize is:

\[
\int_{0}^{\infty} \frac{c^{1-\theta}}{1-\theta} e^{-\rho t} dt
\]

under the following dynamic constraint:

\[
l w + b i = \dot{b} + c
\]

where \(b\) is personal wealth and \(i\) is the rate of return on it.

From the optimality condition and substituting for \(l\) (see Appendix 1) the economy dynamics is defined by:

\[
\dot{\hat{c}} = \frac{\Theta^{-1}}{\alpha} \left(\alpha a \hat{c}^{\frac{1}{\alpha}} - \rho - g \Theta\right)
\]

\[
\dot{\hat{k}} = a \hat{c}^{\frac{1}{\alpha}} - \frac{\hat{k}}{k} - g
\]

where \(a = \left(\frac{1 - \alpha}{\psi}\right)^{\frac{1}{\alpha}} .\)

The linearization around the steady state of the above system implies the following convergence equation (see Appendix 1):

\[
\Delta \log \hat{y}_i = (1 - \alpha) \Delta \log l_i - \left(1 - e^{\hat{c} \hat{r}}\right) \log y_0 + \left(1 - e^{\hat{k} \hat{r}}\right) \left(1 - \alpha\right) \log l_0 e^{\hat{c} \hat{r}} - \left(1 - e^{\hat{k} \hat{r}}\right) \log \psi + \text{Constant}
\]

Noting that the log of the employment rate approximates the unemployment rate \(\log l_i \equiv -u_i\), equation (13) implies a negative relationship between growth and unemployment as well as between growth and the mark up.

It is also possible to substitute for the employment rate to get a convergence equation that is only function of parameters (see Appendix 1):
\[
\Delta \log y_t = - \left(1 - e^{\xi_t L} \right) \log y_0 - \left(1 - e^{\xi_t L} \right) \log \psi + \left(1 - e^{\xi_t L} \right) \Pi + g_t + \left(1 - e^{\xi_t L} \right) \log A_0
\]

Equations (14) and (13) make explicit the negative link between rigidity in the labor market and growth along the transitional.

3. **Empirical results**

   In this section we investigate empirically the relationships between labor market imperfections and growth implied by the model using a data set of up to 18 OECD countries for the period 1960-1990. Finding a good proxy for labor market imperfection and in particular for the mark-up of wage over the reservation wage \(\psi_{i,t}\) is quite a difficult task (see Bean 1994). The variables most commonly used in the literature are the benefit replacement ratio (the ratio of unemployment benefits over the wage) and the union power. However these proxies are not without criticisms (Bean 1994). The unemployment benefits vary across countries by magnitude, duration and eligibility, therefore some sort of discretionary average must be performed. In the case of Italy, some unemployment benefits (the cassa integrazione guadagni) assume peculiar legislative forms and they are sometime included sometime excluded from the OECD measures (see Martin J.P. (1996)). The proxy for union power commonly suggested is the union density (the ratio of unionized workers to the work force) however it captures the potential union power without identifying the extent to which union power is exploited (Bean,1994). As mentioned in section 2, Bean (1994,1994b) suggests that per capita consumption should also influence the mark-up. Therefore we propose an alternative mark-up proxy, the nominal ratio of total compensation per employee to per capita private consumption, that should better capture Bean’s insight as well as be more linked to our theoretical model.

   In the proceeding we test the model by considering the standard mark-up proxies at first. Subsequently we put forth some evidence in favor of our new mark-up proxy and test the model by using it.

3.1 **Results for Standard Proxies**

   Table 1 reports the results of several cross countries growth regressions based on equation (14) where the average annual growth rate of per capita income for the period 1960-1990 (Sources PENN World Table 5.5) is regressed against its initial log level, the log of the average net replacement ratio (LBRR) (sources: OECD (1994)) and the log of the average union density (LUNION) (sources: Layard, Nickell and Jackman (1991) and OECD (1994)). We also control for the accumulation of human and physical capital measured as the log of the average enrollment ratio (LSH) and the log of the average investment-output ratio (LSK) respectively (Sources PENN World Table 5.5). We exclude Spain from some analyses due to the presence of too many missing values in the variable LUNION. We report the results about LBRR with and without Italy for the presence of the high measurement errors mentioned by Martin (1996).

---

3 The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, West-Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, United Kingdom and United States.
As we can notice from Table 1 the variable net replacement ratio is negative and significant at 10% or less in all regressions but in model 1.6 where Italy is included. The variable union density is also always significant at 10% or less and it has the expected negative sign.

Table 1
Dependent variable: average annual growth rate of per capita income (1960-1990)
(White Heteroskedasticity-Consistent Standard Errors in italics)

<table>
<thead>
<tr>
<th></th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.234172</td>
<td>0.143401</td>
<td>0.172226</td>
<td>0.225423</td>
<td>0.127695</td>
<td>0.160498</td>
<td>0.263524</td>
<td>0.161531</td>
<td>0.208461</td>
</tr>
<tr>
<td></td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.037487</td>
<td>0.057157</td>
</tr>
<tr>
<td>LY1960</td>
<td>-0.02188\textsuperscript{a}</td>
<td>-0.02026\textsuperscript{a}</td>
<td>-0.01959\textsuperscript{a}</td>
<td>-0.02166\textsuperscript{a}</td>
<td>-0.02039\textsuperscript{a}</td>
<td>-0.01925\textsuperscript{a}</td>
<td>-0.0249\textsuperscript{a}</td>
<td>-0.02229\textsuperscript{a}</td>
<td>-0.02191\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td>0.003882</td>
<td>0.003764</td>
<td>0.003691</td>
<td>0.004046</td>
<td>0.003685</td>
<td>0.003577</td>
<td>0.004379</td>
<td>0.004081</td>
<td>0.004623</td>
</tr>
<tr>
<td>LBRR</td>
<td>-0.00517\textsuperscript{b}</td>
<td>-0.00356\textsuperscript{c}</td>
<td>-0.00428\textsuperscript{b}</td>
<td>-0.00308\textsuperscript{c}</td>
<td>-0.00259\textsuperscript{b}</td>
<td>-0.00242</td>
<td>-0.00332\textsuperscript{b}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.002108</td>
<td>0.001752</td>
<td>0.001881</td>
<td>0.001779</td>
<td>0.001137</td>
<td>0.001648</td>
<td></td>
<td></td>
<td>0.001539</td>
</tr>
<tr>
<td>LSK</td>
<td>0.011298\textsuperscript{c}</td>
<td>0.0112125\textsuperscript{b}</td>
<td>0.011281\textsuperscript{c}</td>
<td>0.01295\textsuperscript{b}</td>
<td>0.011612\textsuperscript{b}</td>
<td>0.010631\textsuperscript{c}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.005439</td>
<td>0.005019</td>
<td>0.005466</td>
<td>0.005293</td>
<td>0.004978</td>
<td>0.005682</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LSH</td>
<td>0.008014</td>
<td>0.011166</td>
<td>0.009432\textsuperscript{a}</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.007886</td>
<td>0.007029</td>
<td>0.002802</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LUNION</td>
<td></td>
<td></td>
<td>-0.00507\textsuperscript{c}</td>
<td>-0.00494\textsuperscript{b}</td>
<td>-0.0037\textsuperscript{a}</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.002592</td>
<td>0.002071</td>
<td>0.00165</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2-adj</td>
<td>0.783353</td>
<td>0.806307</td>
<td>0.812491</td>
<td>0.765023</td>
<td>0.809736</td>
<td>0.79922</td>
<td>0.782093</td>
<td>0.819782</td>
<td>0.820788</td>
</tr>
<tr>
<td>Obs.</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>17</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>W/o Italy</td>
<td>W/o Italy</td>
<td>w/o Italy</td>
<td>w/o Italy</td>
<td>w/o Spain</td>
<td>w/o Spain</td>
<td>w/o Spain, Italy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a significant at 1%; b significant at 5%; c significant at 10%. All variables but the initial income are average values of the period 1960-1990.

OLS cross section estimates have been criticized on econometric grounds for not controlling for unobservable fixed effects and for the presence of endogenous regressors (Islam (1995), Caselli et alii (1996)). Therefore in the following we check the robustness of the above results testing our model in a panel data context.

Observations for LBRR are available more or less at the start of every decade from 1960 to 1990. We fill in the missing values for the years 1965, 1975 and 1985 by averaging over the preceding observation and subsequent one. We do not try to perform the estimates also using the union density variable since we have not enough observations for a reliable dynamic panel data analysis. We adapt equation (14) and apply the
model to a balanced panel of five-years apart observations\(^{4}\) on 17 OECD countries\(^{5}\), excluding Italy from the analysis for the reasons outlined above.

We estimate the following linear model with time and individual effects:

\[
(15) \quad \log y_{i,t} = \beta_1 \log y_{i,t-1} + \beta_2 \log \psi_{i,t-1} + \lambda_i + \eta_i + \epsilon_{i,t}, \quad i = 1, \ldots, N; \quad t = 2, \ldots, T
\]

where \(y_{i,t}\) and \(y_{i,t-1}\) are respectively the per capita output for country \(i\) at time \(t\) and five years earlier, \(\log \psi_{i,t}\) is the LBBR variable computed as explained above and \(T=7\).

In equation (15), \(v_{i,t} = \lambda_i + \eta_i + \epsilon_{i,t}\) is the "fixed effects" decomposition of the error term in which \(\eta_i\) and \(\lambda_i\) are the country and the time specific effects respectively. The error component \(\epsilon_{i,t}\) is assumed to be serially uncorrelated with zero mean and independently distributed across countries, but heteroskedasticity across time and country is allowed for. Moreover, \(\epsilon_{i,t}\) is assumed to be uncorrelated with the initial condition \(\log y_{i,1}\), for \(t=2, \ldots, T\), and with the individual effects \(\eta_i\) for any \(t\).

The model (15) outlined above is a dynamic panel data regression model where the lagged dependent variable and (possibly) the \(\psi_{i,t}\) variable are correlated with the individual effect. Moreover, the \(\psi_{i,t}\) variable is assumed to be strictly exogenous with respect to \(\epsilon_{i,t}\).

As far as the estimation of (15) is concerned, consistent and efficient estimates can be obtained applying the generalized method of moments (GMM) framework in the form recently proposed by Arellano and Bover (1995) and Blundell and Bond (1998). We adopt this method to try to overcome the criticisms that arise when using either of the two approaches: the traditional cross-countries growth regressions and the more recent dynamic panel data models. As mentioned above, the OLS cross-countries approach has been criticized on econometric grounds for not controlling for unobservable fixed effects. The standard dynamic panel data approach, on the other hand, has been blamed for losing between-countries information when the individual effects are eliminated by differencing the variables (Barro (1997), Temple (1999)).

The linear system GMM estimator permits to efficiently combine information from the equations in levels with the set of orthogonality conditions holding for the equations in first differences. In so doing it is possible to retain part of the between-countries variation besides controlling for individual heterogeneity.

Let us rewrite the level equations (15) in first differences:

\[
(16) \quad \Delta \log y_{i,t} = \beta_1 \Delta \log y_{i,t-1} + \beta_2 \Delta \log \psi_{i,t-1} + \lambda_i^* + \Delta \epsilon_{i,t}, \quad i = 1, \ldots, N; \quad t = 3, \ldots, T
\]

where \(\Delta \log y_{i,t} = \log y_{i,t} - \log y_{i,t-1}\), and analogously for the other variables, and \(\lambda_i^*\) stands for the transformed time effect.

\(^{4}\) The use of quinquennial averages is an open issue, see Temple (1999) on this point.

\(^{5}\) Limitations in finding data on the benefit replacement ratio prevent us to enlarge the sample. In the following we will use estimators that require \(N\) large for reliability of the estimates. One therefore should be careful in interpreting the results obtained.
Note that given the assumptions above, the following orthogonality conditions relative to country \( i \) can be used for estimating equations (16):

\[
\begin{align*}
E(\log y_{i,t-s}, \Delta e_{i,t}) &= 0 \quad \text{for } s \geq 2, t = 3, \ldots, T \\
E(\log \psi_{i,t-s}, \Delta e_{i,t}) &= 0 \quad \text{for any } s, t = 3, \ldots, T.
\end{align*}
\]

These moment conditions qualify the levels of \( \log y_{i,t} \) lagged two periods and earlier and the forward, current and lagged levels of \( \log \psi_{i,t} \) as instruments at time \( t \) for the first differenced regressors\(^6\) appearing in (16) and give rise to the GMM difference estimator proposed by Arellano and Bond (1991)\(^7\). Arellano and Bover (1995) and Blundell and Bond (1998) argue that often additional a priori information relative to the equations in levels is available which may be used to improve the GMM difference estimator. Furthermore, their approach allows to estimate the parameters of time invariant regressors that disappear when one uses only first-difference estimation.

Basically, an instrument for the level equations is a variable uncorrelated with the individual effects \( \eta_i \) as well as with the error term \( \varepsilon_{i,t} \). In their papers the authors propose to use lagged differences as possible instruments for equations in levels. In particular Blundell and Bond (1998) show by Monte Carlo simulations and asymptotic variance calculations that this extended GMM estimator improves dramatically the precision of the estimates with respect to GMM difference estimator and greatly reduces the finite sample bias too. This is particularly relevant for dynamic panels where the autoregressive parameter is relatively large and \( T \) is relatively small\(^8\).

We consider the following additional orthogonality conditions:

\[
E(\Delta \log y_{i,t-1}(\eta_i + \varepsilon_{i,t-1})) = 0; \quad E(\Delta \log \psi_{i,t-1}(\eta_i + \varepsilon_{i,t-1})) = 0
\]

for \( t = 3, \ldots, T \), which are automatically satisfied under the hypotheses of stationarity of the model and constant correlation between the mark up variable and the individual effects\(^9\).

In Table 2 both the difference GMM and the system GMM estimates for the panel of 17 OECD countries are reported\(^10\).

As suggested by Blundell and Bond (1998)\(^11\), we use the one-step version of the GMM estimators, robust to time and country heteroskedasticity\(^12\). Moreover, all the estimates are obtained using the instruments \( \log y_{i,t} \).

\(^{\text{6}}\) Notice that in (38) the regressor \( \Delta y_{i,t-1} \) is correlated with the error \( \Delta e_{i,t} \). Moreover, under the assumption of no serial correlation, the differenced errors will be MA(1) with a unit root, so that first order serial correlation in the differenced residuals is expected, but no second-order serial correlation.

\(^{\text{7}}\) See Caselli et al. (1996) for an application of the method to testing for "growth convergence".

\(^{\text{8}}\) In this case lagged levels of the variables provide weak instruments for first differences and the GMM difference estimator performs poorly.

\(^{\text{9}}\) That is clearly a weaker condition than requiring that the levels of mark up to be uncorrelated with the individual effects. Moreover, a Hausman test can be performed to test for the validity of these extra-instruments.

\(^{\text{10}}\) See the papers cited above for a formal presentation of the system GMM estimator. All estimates are computed using DPD98 for GAUSS, kindly provided by Prof. M. Arellano. Time dummies are used for modelling the time effects \( \lambda_t \).

\(^{\text{11}}\) They found that the asymptotic \( t \)-tests based on the two-step GMM estimators can be seriously misleading, and tend to reject too frequently. Things worsen, in the case of heteroskedastic errors.

\(^{\text{12}}\) See the Arellano and Bond (1998) DPD98 manual for details.
lagged up to 4 periods in order to reduce finite sample biases deriving from having too much instruments relative to the cross-sectional sample size\textsuperscript{13}. For the same reason we use only the values of $\log y_{t,s} = -2, ..., 0, ..., 2$ as instruments for the differenced equations at time $t$.

In Table 2, column 2.1, we report the difference GMM estimates based on the subset of the moment conditions (17) outlined above. Moreover, we report also some tests for the correct specification of the model. We compute the M1 and M2 tests for first-order and second-order serial correlation in first-differenced residuals and the Sargan test of the over-identifying restrictions. No sign of second order correlation is encountered and the Sargan test is highly not rejected confirming that all the instruments are accepted. As in previous empirical analysis, the (log) per capita output lagged five years is highly significant with an estimated regression coefficient of 0.65. The variable benefit replacement ratio enter the equation negatively but not much significantly. The large standard error obtained for the variable LBRR may be attributed to a low efficiency of the estimates as consequence of the fact that we used too few instruments for LBRR. Therefore in column column 2.2 we re-estimate the model using all the available instruments for the LBRR variable. We notice that the standard error is extremely low in this case so that the variable LBRR enter the equation significantly and negatively.

In order to accomplish both a reduction of finite sample biases and an improvement in the precision of the estimates we now apply the system GMM estimator which uses the additional instruments (18). In particular we estimate the model by using the same restricted instruments of model 1.1 for the differenced equations and the additional instruments (18) for the level equations.

The system GMM estimates are reported in column 2.3 together with the Hausman specification test and the Difference Sargan test for the validity of the extra-instruments (18). Following Arellano and Bond (1991) and Arellano (1993) the Hausman test is computed as a Wald test and it is asymptotically distributed as a $\chi^2(2)$ under the null. Both the Hausman test and the Difference Sargan test confirm that the level equations instruments are highly not rejected. The results of the estimates are encouraging since the LBRR coefficient results to be highly significant and with the same magnitude of the coefficient estimated in column 2.2.

\textsuperscript{13} See Kiviet (1995) or Alonso-Borrego and Arellano (1999).
Table 2
One-step robust GMM estimates; the dependent variable is the log of per capita output \( \log y_{i,t} \); \( \log y_{i,t-1} \) is the LBRR variable; 17 countries, 85 observations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>2.1</th>
<th>2.2</th>
<th>2.3</th>
<th>2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log y_{i,t-1} )</td>
<td>0.658387 (0.058411)</td>
<td>0.690320 (0.052151)</td>
<td>0.679943 (0.093503)</td>
<td>0.759537 (0.069574)</td>
</tr>
<tr>
<td>( \log \psi_{i,t-1} )</td>
<td>-0.005986 (0.010283)</td>
<td>-0.012263 (0.006912)</td>
<td>-0.012123 (0.005771)</td>
<td>-0.022948 (0.009445)</td>
</tr>
<tr>
<td>( \log sk_{i,t} )</td>
<td></td>
<td></td>
<td></td>
<td>0.032244 (0.053140)</td>
</tr>
<tr>
<td>( \log sh_{i,t-1} )</td>
<td></td>
<td></td>
<td></td>
<td>0.042546 (0.064331)</td>
</tr>
</tbody>
</table>

M1: -2.22 -2.28 -2.40 -2.43  
M2: 0.57 0.47 -0.47 0.36  
Sargan (df): 9.94 (32), 8.65 (45), 8.51 (42), 9.18 (66)  
Diff-Sargan (df): -1.43 (10), 1.75 (18)  
Hausman (df): 4.18 (2), 13.29 (4)  

Notes.
1. Time dummies are included in all models.
2. Model 2.1 and 2.2 are estimated by difference GMM estimator. Model 2.3 and 2.4 are estimated by system GMM estimator. Except for Model 2.2, the instruments used in each equation (where available and if the corresponding regressor is included into the model) are
   **Differenced equations:** \( \log y_{i,t-2} \), \( \log y_{i,t-3} \), \( \log y_{i,t-4} \); \( \log \psi_{i,t+2} \), \( \log \psi_{i,t+3} \), \( \log \psi_{i,t+4} \); \( \log sk_{i,t-2} \), \( \log sk_{i,t-3} \), \( \log sk_{i,t-4} \); \( \log sh_{i,t-2} \), \( \log sh_{i,t-3} \), \( \log sh_{i,t-4} \).
   **Level equations:** \( \Delta \log y_{i,t-1} \), \( \Delta \log \psi_{i,t-1} \), \( \Delta \log sk_{i,t-1} \), \( \Delta \log sh_{i,t-1} \).
3. Model 2.2 uses all the available values of \( \log \psi_{i,t} \) as instruments for the differenced equations.
4. Asymptotic robust standard errors are reported in parentheses.
5. M1 and M2 tests for first-order and second-order serial correlation in the differenced residuals are asymptotically distributed as a N(0,1) under the null of no serial correlation.
6. The Sargan and Difference Sargan tests of over-identifying restrictions, computed from two-step estimates, are asymptotically distributed as a \( \chi^2 \) under the null of instruments validity and extra-instruments validity respectively, with degrees of freedom reported in parentheses.
7. The Hausman specification test, computed as a Wald test from one-step robust estimates, is asymptotically distributed as a \( \chi^2 \) under the null of instruments validity for the level equations, with degrees of freedom reported in parentheses.
In order to verify the robustness of this result, we add to the model some regressors suggested by the convergence literature (see Temple (1999) for a survey) and we report the results of the estimates in Table 2, Model 1.c. The Model 1.c is specified as follows:

\[
\log y_{i,t} = \beta_1 \log y_{i,t-1} + \beta_2 \log y_{i,t-1} + \beta_3 \log sk_{i,t} + \beta_4 \log sh_{i,t-1} + \lambda_i + \eta_i + \epsilon_{i,t} \\
i = 1,...,N; t = 2,....,T
\]

where \(sh_{i,t}\) is the total enrolment in high school at time \(t^{14}\), while \(sk_{i,t}\) is the quinquennial average investment over GDP ratio.

All additional regressors included at time \(t\) are treated as potentially endogenous with respect to the error component \(\epsilon_{i,t}\). Furthermore, constant correlation across time between the individual effects \(\eta_i\) and each regressor is also assumed, so that, in addition to conditions (17) and (18), the following orthogonality conditions hold:

\[
E(\log sk_{i,t-s}\Delta \epsilon_{i,t}) = E(\log sh_{i,t-s}\Delta \epsilon_{i,t}) = 0
\]

and

\[
E(\Delta \log sk_{i,t-s}\omega_{i,t}) = E(\Delta \log sh_{i,t-s}\omega_{i,t}) = 0
\]

for \(t = 3,...,T\) and \(s \geq 2\), where \(\omega_{i,t} = \eta_i + \epsilon_{i,t}\), for the differenced equations and for the level equations respectively. In the estimates we utilize the above orthogonality conditions for the differenced equations up to \(s = 4\).

Looking at the results of the system GMM estimates, the proxy for mark up continues to affect negatively the per capita output, while both the physical and human capital accumulations rates enter the equation positively but not significantly. We notice that the Difference Sargan test does accept the extra-instruments, while the Hausman test does not. However, the variables \(\log sk_{i,t}\) and \(\log sh_{i,t-1}\) are jointly insignificant also applying the difference GMM estimator (here not reported), computed by using orthogonality conditions of the same type of those used in model 2.2, so that the results outlined above for model 2.3 (and model 2.2) are confirmed.

### 3.2 Results for an alternative proxy

In this subsection we propose an alternative proxy for the mark-up \(\psi_{i,t}\) and present the results obtained for our panel of OECD countries. Equation (7) suggests a simple way to measure the mark-up \(\psi_{i,t}\): the ratio of per capita wage over per capita consumption. We compute it as the log of total compensation per employee over per capita private consumption (LWC), all at current prices (sources OECD data), averaged according

---

14 The data on total enrolment in high school, from Barro and Lee(1993), present some missing values that we have filled in by linear interpolation. They are available for the beginning of each quinquennium and this explains why the variable is inserted as a lagged regressor.
to the time frequency used in the analyses. Indeed the proxy we propose is likely to suffer from endogeneity problems, which we try to overcome by instrumenting for it in the subsequent panel data analysis.

Among the pros of our choice, besides testing one of our model’s assumptions, is the fact of capturing the hints of Bean (1994) about the importance of per capita consumption in setting the reservation wage. The suggested proxy appears to be positively cross countries correlated with the more utilized measure of labor market imperfections (see Table 3) such as the replacement ratio, the unemployment benefit duration, the union density and the OECD index of worker protection legislation.

Table 3

Cross Countries Correlations relative to averaged values of the variables over the period 1960-90.

<table>
<thead>
<tr>
<th></th>
<th>LWC</th>
<th>LBRR</th>
<th>LUNION</th>
<th>Log Benefit Duration</th>
<th>Log Index of Protection</th>
</tr>
</thead>
<tbody>
<tr>
<td>LWC</td>
<td>1.000000</td>
<td>0.415379</td>
<td>0.272617</td>
<td>0.264322</td>
<td>0.207711</td>
</tr>
<tr>
<td>LBRR</td>
<td>0.415379</td>
<td>1.000000</td>
<td>0.327887</td>
<td>0.713146</td>
<td>0.397880</td>
</tr>
<tr>
<td>LUNION</td>
<td>0.272617</td>
<td>0.327887</td>
<td>1.000000</td>
<td>0.470454</td>
<td>0.416434</td>
</tr>
<tr>
<td>Log of Benefit Duration</td>
<td>0.264322</td>
<td>0.713146</td>
<td>0.470454</td>
<td>1.000000</td>
<td>0.423209</td>
</tr>
<tr>
<td>Log of Index of Protection</td>
<td>0.207711</td>
<td>0.397880</td>
<td>0.416434</td>
<td>0.423209</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

All variables but the Index of Protection (Oecd,1994,pg.74) are average value of the period. Italy and Spain are not included in the sample.

The results of the cross countries regressions are reported in Table 4. The new mark-up proxy LWC results to be negatively correlated with long run growth in all models but significant at 10% only in model 4.1. In model 4.3 the LBRR variable turns out to be not significant, an outcome that suggests the presence of multicollinearity problems given the results in Table 1. Since in model 4.4 LUNION is still significant we suspect that multicollinearity affects mainly the two variables LBRR and LWC (the most correlated variables in Table 3). It is worth remembering that the results about LWC might be misleading in this framework due to not only multicollinearity but also to omitted individual effects and endogeneity of the regressors.
Table 4
Dependent variable: average annual growth rate of per capita income (1960-1990)
(White Heteroskedasticity-Consistent Standard Errors in italics)

<table>
<thead>
<tr>
<th></th>
<th>4.1</th>
<th>4.2</th>
<th>4.3</th>
<th>4.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.214</td>
<td>0.126</td>
<td>0.149</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>0.030</td>
<td>0.037</td>
<td>0.044</td>
<td>0.071</td>
</tr>
<tr>
<td>LY60</td>
<td>-0.020&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.019&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.019&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.022&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>0.003</td>
<td>0.003</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>LWC</td>
<td>-0.017&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-0.009</td>
<td>-0.005</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>0.009</td>
<td>0.008</td>
<td>0.008</td>
<td>0.01</td>
</tr>
<tr>
<td>LBRR</td>
<td>-0.003</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>LSK</td>
<td>0.010&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.010&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.010&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>0.004</td>
<td>0.004</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>LSH</td>
<td>0.010</td>
<td>0.006</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.007</td>
<td>0.008</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>LUNION</td>
<td>0.003&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2-adj.</td>
<td>0.77</td>
<td>0.78</td>
<td>0.79</td>
<td>0.78</td>
</tr>
<tr>
<td>Obs.</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>W/o Italy W/o Italy W/o Italy w/o Italy and Spain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> significant at 1%; <sup>b</sup> significant at 5%; <sup>c</sup> significant at 10%.

The panel data analysis should reach more reliable conclusions. In Table 5 we report the results of the one-step robust system GMM estimates, where \( \log y_{i,t} \) is now the quinquennially averaged LWC. The instruments used to take account of its endogeneity are reported in note n. 3 of the Table 5. For the model in column 5.1 (and also in column 5.3) the Wald statistic on the joint significance of time dummies result to be just a little inside the 5% acceptance interval<sup>15</sup>. Therefore we perform the analyses both including time dummies (column 5.1 and 5.3) and including a constant term only (column 5.2 and 5.4). As we can see from the estimates our proposed proxy for mark-up LWC always affects negatively the per capita output. The results of Table 2 concerning the LBRR variable are confirmed in model 5.1 and 5.3 but they do not hold when as suggested by the Wald test the time dummies are dropped. The human and physical capital accumulation rates are never significant with and without time dummy variables.

The Difference Sargan test does accept the extra-instruments in all specifications while the Hausman test does accept the null hypothesis only for the models with a constant term only. Therefore according to all the specification tests we should consider the model 5.4 as the preferred model.

---

<sup>15</sup> For the model 5.1 the Wald statistic is \( \chi^2(5) = 10.81 \) and for the model 5.3 the value is \( \chi^2(5) = 10.95 \).
Nevertheless given the results in Table 2 about the LBRR variable and the likely collinearity between LBRR and LWC we tend to consider both variables as important determinants of long run growth.

Table 5

One-step robust system GMM estimates; the dependent variable is the log of per capita output $\log y_{i,t}$; $\log y_{i,t-1}$ is the LBRR variable; $\log y_{i,t}$ is the LWC variable.

<table>
<thead>
<tr>
<th>Variable</th>
<th>5.1</th>
<th>5.2</th>
<th>5.3</th>
<th>5.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log y_{i,t-1}$</td>
<td>0.816257</td>
<td>0.849568</td>
<td>0.802884</td>
<td>0.832797</td>
</tr>
<tr>
<td></td>
<td>(0.059143)</td>
<td>(0.048116)</td>
<td>(0.069591)</td>
<td>(0.025089)</td>
</tr>
<tr>
<td>$\log y_{i,t}$</td>
<td>-0.177334</td>
<td>-0.177351</td>
<td>-0.176675</td>
<td>-0.212444</td>
</tr>
<tr>
<td></td>
<td>(0.086356)</td>
<td>(0.068887)</td>
<td>(0.072712)</td>
<td>(0.066101)</td>
</tr>
<tr>
<td>$\log sk_{i,t}$</td>
<td>-0.020913</td>
<td>0.029425</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.077545)</td>
<td>(0.049890)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log sh_{i,t-1}$</td>
<td>0.046706</td>
<td>0.002129</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.062605)</td>
<td>(0.030722)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log y_{i,t-1}$</td>
<td>-0.017979</td>
<td>-0.010761</td>
<td>-0.014347</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.009573)</td>
<td>(0.010391)</td>
<td>(0.005141)</td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>-2.33</td>
<td>-2.80</td>
<td>-2.40</td>
<td>-2.73</td>
</tr>
<tr>
<td>M2</td>
<td>0.46</td>
<td>1.14</td>
<td>0.54</td>
<td>1.21</td>
</tr>
<tr>
<td>Sargan (df)</td>
<td>10.08</td>
<td>10.24</td>
<td>10.15</td>
<td>13.02</td>
</tr>
<tr>
<td></td>
<td>(78)</td>
<td>(78)</td>
<td>(54)</td>
<td>(28)</td>
</tr>
<tr>
<td>Diff.-Sargan (df)</td>
<td>0.59</td>
<td>0.16</td>
<td>-0.71</td>
<td>-1.72</td>
</tr>
<tr>
<td></td>
<td>(22)</td>
<td>(22)</td>
<td>(14)</td>
<td>(9)</td>
</tr>
<tr>
<td>Hausman (df)</td>
<td>20.97</td>
<td>8.16</td>
<td>42.52</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(5)</td>
<td>(5)</td>
<td>(3)</td>
<td>(2)</td>
</tr>
<tr>
<td># countries</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Obs.</td>
<td>85</td>
<td>85</td>
<td>85</td>
<td>85</td>
</tr>
</tbody>
</table>

Notes.
1. Time dummies are included in models 5.1 and 5.3; only a constant term is included in models 5.2 and 5.4.
2. The proxies for the mark-up variable $\psi_{i,t}$ and $\psi_{i,t-1}$ are measured as total compensation per employee over per capita private consumption quinquennially averaged and the average net replacement ratio respectively.
3. The instruments used in each equation (where available and if the corresponding regressor is included into the model) are

**Differenced equations:** $\log y_{i,t-2}$, $\log y_{i,t-3}$, $\log y_{i,t-4}$; $\log y_{i,t-1}$, $\log y_{i,t-2}$, $\log y_{i,t-3}$, $\log y_{i,t-4}$; $\log y_{i,t+2}$, ..., $\log y_{i,t-2}$

$\log sk_{i,t-2}$, $\log sk_{i,t-3}$, $\log sk_{i,t-4}$; $\log sh_{i,t-2}$, $\log sh_{i,t-3}$, $\log sh_{i,t-4}$.

**Level equations:** $\Delta \log y_{i,t-1}$, $\Delta \log y_{i,t-1}$, $\Delta \log y_{i,t-1}$, $\Delta \log y_{i,t-1}$, $\Delta \log sk_{i,t-1}$, $\Delta sh_{i,t-1}$.

4. Asymptotic robust standard errors are reported in parentheses.
7. M1 and M2 tests for first-order and second-order serial correlation in the difference residuals are asymptotically distributed as a N(0,1) under the null of no serial correlation.
8. The Sargan and Difference Sargan tests of over-identifying restrictions, computed from two-step estimates, are asymptotically distributed as a $\chi^2$ under the null of instruments validity and extra-instruments validity respectively, with degrees of freedom reported in parentheses.
9. The Hausman specification test, computed as a Wald test from one-step robust estimates, is asymptotically distributed as a $\chi^2$ under the null of instruments validity for the level equations, with degrees of freedom reported in parentheses.
Conclusions

In this paper we explore both theoretically and empirically the possible effects of an imperfect labor market on growth. We show that considering an imperfect labor market with a monopolistic union in a neoclassical growth model the degree of labor market imperfection does affect the growth rate along the transitional path. In the empirical part of the paper we test the model implications on a panel of 18 OECD countries. The analysis is performed in a standard cross countries growth regression framework as well as in a panel data context applying the system GMM estimator proposed by Arellano and Bond (1995) and Blundell and Bond (1998). We find that the standard proxies of mark-up suggested in the literature, namely the unemployment replacement ratio and the union power, have negative long run growth effects.

We propose an alternative mark-up proxy that should capture the Bean’s (ibid., 1994) insight and implicitly tests for one of our model’s assumptions: the ratio of per capita wage over per capita consumption. This variable appears to influence negatively and significantly the growth of output.

Both the theoretical and empirical analyses we present imply that labor market imperfections have long run growth effects. In turn these findings suggest that the costs of these imperfections are likely to be greatly underestimated.
Appendix 1
The optimality conditions of the consumer problems are:

A.1 \[ \dot{c} = c \theta^{-1} (i - \rho) \]

A.2 \[ \lim_{t \to \infty} a(t) \frac{d}{dc} \left( \frac{e^{\theta t}}{1 - \theta} \right) e^{-\rho t} = 0 \]

Ignoring capital depreciation the rate of return of wealth is equal to the marginal capital productivity:

A.3 \[ i = \alpha k^{\alpha - 1} l^{1 - \alpha} A^{1 - \alpha} \]

Moreover, in aggregate terms net wealth is equal to physical capital and from A.3 and the production function we can write equations A.1 and (10) as:

A.4 \[ \frac{\dot{c}}{c} = \theta^{-1} (\alpha k^{\alpha - 1} l^{1 - \alpha} A^{1 - \alpha} - \rho) \]

A.5 \[ \dot{k} = k^{\alpha} l^{1 - \alpha} A^{1 - \alpha} - c \]

We assume the following standard law of motion for the technological parameter:

A.6 \[ A(t) = A(0) e^{\xi t} \]

Given A.6, equations A.4 and A.5 form a non autonomous system. However we can rewrite the system of differential equations as an autonomous system:

A.7 \[ \frac{\dot{c}}{c} - g = \frac{\dot{c}}{\widehat{c}} = \theta^{-1} (\alpha \hat{k}^{\alpha - 1} l^{1 - \alpha} - \rho - g \theta) \]

A.8 \[ \frac{\dot{k}}{k} - g = \frac{\dot{k}}{\widehat{k}} = k^{\alpha - 1} l^{1 - \alpha} - \frac{\widehat{c}}{\widehat{k}} - g \]

Equations A.7, A.8 and (8) fully define the economy.

Substituting the employment rate \( l \) in A.7 and A.8 we obtain:

A.9 \[ \frac{\dot{c}}{\widehat{c}} = \theta^{-1} (\alpha a \frac{\alpha - 1}{\alpha} \frac{\alpha - 1}{\alpha} - \rho - g \theta) \]

A.10 \[ \frac{\dot{k}}{\widehat{k}} = \alpha \frac{\alpha - 1}{\alpha} - \frac{\widehat{c}}{\widehat{k}} - g \]

where \( a = \left( \frac{1 - \alpha}{\psi} \right)^{\frac{1 - \alpha}{\alpha}}. \)

Solving for the steady state values of per capita consumption, per capita capital and the unemployment rate we obtain:
A.11 \[ \hat{c}^* = \left( \frac{\alpha}{\rho + g\theta} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{1-\alpha}{\psi} \right) \]

A.12 \[ \hat{k}^* = \left( \frac{\alpha}{\rho + g(\theta - \alpha)} \right) \left( \frac{\alpha}{\rho + g\theta} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{1-\alpha}{\psi} \right) \]

A.13 \[ l^* = \left( \frac{\rho + g\theta}{\rho + g(\theta - \alpha)} \right) \left( \frac{1-\alpha}{\psi} \right) \]

It is worth noticing that A.11, A.12 and A.13 imply that any increase in the markup over the reservation wage (\( \psi \)) affects negatively the steady states values of capital and consumption as well as the unemployment rate. This is the mechanism by which labor market rigidity affects growth along the transition. In the neoclassical model, the growth of the economy along the transitional depends positively on the distance of the economy from the steady state. An increase in the markup parameter lowers the steady state values of output determinants thus lowering the economy growth rate.

The linearization around the steady state of the above dynamic system implies the following:

(A.14) \[ \dot{\log c} = \left( \frac{\alpha - 1}{\alpha} \right) \left( \frac{\rho + g\theta}{\theta} \right) \left( \log \hat{c} - \log \hat{c}^* \right) \]

(A.15) \[ \dot{\log k} = \left[ g - \frac{1}{\alpha} \left( \frac{\rho + g\theta}{\alpha} \right) \right] \left( \log \hat{c} - \log \hat{c}^* \right) + \left( g + \left( \frac{\rho + g\theta}{\alpha} \right) \right) \left( \log \hat{k} - \log \hat{k}^* \right) \]

The solution of the above system is the following:

(A.16) \[ \dot{\log c} = \log \hat{c}^* + e^{\xi_{11}} B_1 \]

(A.17) \[ \dot{\log k} = \log \hat{k}^* + e^{\xi_{22}} B_2 + e^{\xi_{32}} B_3 \]

where:

\[ \xi_{11} = \left( \frac{\alpha - 1}{\alpha} \right) \left( \frac{\rho + g\theta}{\theta} \right) ; \xi_{22} = \left( g + \left( \frac{\rho + g\theta}{\alpha} \right) \right) ; B_1 = \left( \log \hat{c}_0 - \log \hat{c}^* \right) ; B_2 = \left( \log \hat{k}_0 - \log \hat{k}^* \right) \]

and \( B_3 = 0 \) to ensure the existence of the stable path.

We can write the following convergence equation:

(A.18) \[ \Delta \log \hat{y}_t = (1-\alpha) \Delta \log l_t - (1-e^{\xi_{11}}) \log y_0 + \left( 1-e^{\xi_{22}} \right) \left( 1-\alpha \right) \Delta \log \epsilon_{0} e^{\xi_{32}} - \left( 1-e^{\xi_{32}} \right) \log \psi + \text{Constant} \]

Noting that the log of the employment rate approximates the unemployment rate (\( \log l_t \equiv -u_t \)), equation (A.18) implies a negative relationship between growth and unemployment as well as between growth and the mark up. Nevertheless from equations (8), (A.16) and (A.17) it is possible to express the log of employment rate as:
(A.19) \[ \log l_t = e^{h_{t,t}} \log l_0 + (1 - e^{h_{t,t}}) \log l^* \]

Considering (A.18) in levels and substituting for the current employment rate from (A.19) after some algebra one gets:

(A.20) \[ \log \hat{y}_t = e^{h_{t,t}} \log y_0 + (1 - e^{h_{t,t}}) \log \hat{y}_t^* \]

and from (A.20), (A.12), (A.13) and (A.20):

(A.21) \[ \log \hat{y}_t = e^{h_{t,t}} \log \hat{y}_0 - (1 - e^{h_{t,t}}) \log \psi + (1 - e^{h_{t,t}}) \Pi \]

where \[ \Pi = \frac{\alpha}{1-\alpha} \log \alpha - \log (\rho + g(\theta - \alpha)) + \log (1 - \alpha) + \frac{1 - 2\alpha}{1 - \alpha} \log (\rho + g \theta) \]

Subtracting the log of income at time zero from both sides and expressing the equation in observable variables we get:

(A.22) \[ \Delta \log y_t = - (1 - e^{h_{t,t}}) \log y_0 - (1 - e^{h_{t,t}}) \log \psi + (1 - e^{h_{t,t}}) \Pi + g t + (1 - e^{h_{t,t}}) \log A_0 \]

Equations (A.22) and (A.18) implies a negative link between rigidity in the labor market and growth along the transitional.
Cambridge MA.


Phelps E.S. 1968 “Money –Wage Dynamics and labor market equilibrium”, *Journal of Political Economy*, 76,678-711


